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Phase Noise Analysis of a Modified Cross Coupled Oscillator

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***Abstract*— This paper presents a rigorous theoretical phase noise analysis of a modified cross coupled oscillator. A closed-form formula is derived of the phase noise in the 1/f ^{2} region. To verify the derived results, the results are validated against the simulations and illustrate good matches. The overall phase noise error has less than 3 dB error over the offset frequencies from the carrier. The oscillator is designed in a 0.18um CMOS process at 2.3GHz with the phase noise of about 125dBc/Hz at offset frequency of 1MHz while it draws 1.3 mA from 1.8V supply.**

Keywords—oscillator, phase noise

I. Introduction

In the last few decades, many studies have been undertaken to model comprehensively the nature of phase noise in electrical LC oscillators since the oscillators are nonlinear and produce large signals, as well as the process of conversion of noise into the phase noise is not constant but varies with time over one oscillation period. Many of the phased-noise formula are more complicated than Leeson's experimental formulation [1]. Commonly, two models are employed widely in the last two decades.

The first model provided by Demir *et al.,* which is rooted in Kartner [2], is very precise and able to apply to any oscillator such as optic oscillators. In this model the perturbation is decomposed into a phase deviation component and an additive component termed orbital deviation by Demir [3]. Although this model is extremely accurate and used in modern commercially available circuit simulators such as Spectrum RF [4], the mathematical model is very complicated and does not allow the designer physical insight into phase noise generation mechanism.

The second model, introduced by Hajimiri and Lee [5], is a linear time-variant (LTV) model applied only on electrical oscillators. It uses a time-dependent transfer function called an impulse sensitivity function (ISF). ISF indicates how much phase shift stems from exerting a unit current impulse, which is the ratio between the phase shift caused by an instantaneous current perturbation and the amount of injected electric charge. Two impulse response functions are defined for any noise source in this model, which are concerned with amplitude and phase perturbations. The impulse response related to the amplitude perturbations is routinely of a little interest because the amplitude phase is eliminated by the amplitude-limiting mechanism. By contrast, the impulse response corresponding to the phase perturbations has significant interest since the phase noise cannot be omitted by the same technique. Although this model can capture the time variant nature of electrical oscillators and help designers obtain better insight into phase noise generation mechanism, this model fails to predict phase noise once the perturbation is injected whose frequency is very close to the oscillation frequency. As a result, the model is not able to predict injection-locking phenomena [6].

In this paper, the phase noise of the proposed oscillator in [7], represented in Fig. 1, is theoretically analyzed according to Hajimiri's model and verified by simulations. The roles of M_3, M_4, M_5, and M_6 in Fig.1 to improve phase noise are explained in [7]. Therefore, only the theorical calculation of phase noise is provided in this paper. since the theorical calculation of the phase noise is complicated, all the necessary steps are provided in the following sections. This paper is organized as follows. Section II provides the oscillation amplitude, the effective ISFs of the tank and transistors along with a closed-form formula of the phase noise of the proposed oscillator. The simulations and verifications of the theorical calculations in Section II are presented in Section III. The conclusion is in Section IV.

II. Phase Noise Analysis In The Proposed Oscillator

A. Oscillation Amplitude

To derive analytically the phase noise in a LC oscillator, the oscillation amplitude in a large signal analysis should be first calculated. This was rigorously done in [8] and is repeated below

$$V_{out}^{+} = 2 I_B R_{\tan k}\left[1-(\frac{n}{2})\right] \quad (1)$$

Where n is $C_1/(C_1+C_2)$, I_B is the bias current of M_1 or M_2, and R_{tank} is the tank resistance.

B. Effective ISF

Based on Hajimiri's theory, the phase noise $L(\Delta\omega)$ at an offset frequency $\Delta\omega$ from the carrier induced by a white current noise source i_n^2 in a harmonic oscillator is given by [5],[9], and [10],

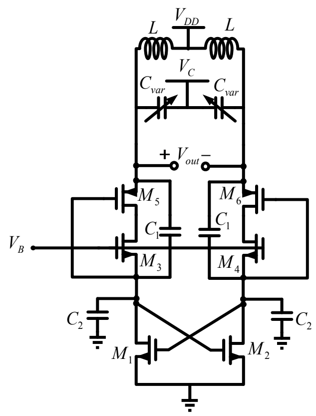

Fig. 1. The proposed oscillator

$$L(\Delta\omega) = 10\log\left(\frac{\Gamma_{rms}^2}{q_{\max}^2} \cdot \frac{\overline{i_n^2}/\Delta f}{2\Delta\omega^2}\right) \tag{2}$$

Where q_{max} is the maximum charge swing across the tank capacitance, $i_n^2/\Delta f$ is the white current noise power spectral density, Γ^2_{rms} is effective ISF. (2) can be simplified by,

$$\Gamma_{rms}^2 \frac{\overline{i_n^2}}{\Delta f} = \frac{1}{2\pi}\int_0^{2\pi} \Gamma^2(\varphi) \frac{\overline{i_n^2}(\phi)}{\Delta f} d\phi \tag{3}$$

Therefore, (2) can be written,

$$L(\Delta\omega) = 10\log\left(\frac{\sum_i N_{L,i}}{2\Delta\omega^2 C^2 V^2}\right) \tag{4}$$

Where $N_{L,i}$ is

$$N_i = \frac{1}{T_0}\int_0^{T_0} \Gamma_i^2(t)\overline{i_{n,i}^2(t)}dt \tag{5}$$

In (5), T_o is the oscillation period and i is the *ith* device. $i_{n,i}^2(t)$ can be either stationary or cyclo-stationary.

C. Tank Effective ISF

If the voltage across the tank is $A\cos(\omega t)$, the tank ISF corresponding to its current noise source, presented in Fig. 2, is a sinusoid in quadrature with the tank voltage, which its magnitude is inversely proportional to the number of the resonators (N) in the oscillator [11]. Therefore, it can be given by,

$$\Gamma_{\tan k} = \frac{-1}{N}\sin(\omega_0 t) = \Gamma_R \tag{6}$$

The square rms value of Γ_R is

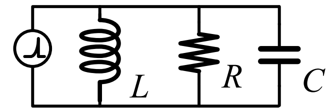

Fig. 2. The tank circuitry to calculate its ISF

$$\Gamma_{R,rms}^2 = \frac{1}{2}\int_{-\pi}^{\pi} \Gamma_R^2(\phi)d\phi = \frac{1}{2N^2} \tag{7}$$

Where the angle $0 \le \varphi \le 2\pi$ is employed rather than $\omega_0 t$. Note that N is one for single-ended oscillators and two for the differential oscillators.

D. Effective ISFs of Transistors

Since the proposed oscillator only works in the differential mode and in order to simplify the calculations in the effective ISFs for the active devices, the half circuit, represented in Fig. 3, is considered. Besides, the transistor ISFs are related to the tank ISF to make the calculation easy as well.

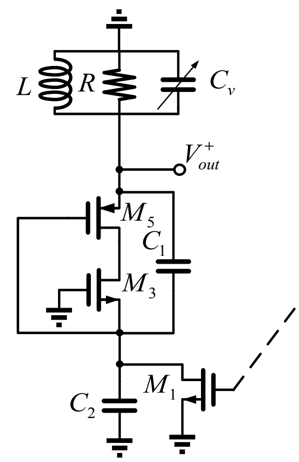

Fig. 3. The half circuit of the proposed oscillator

The ISF for M5, Γ_{M5}, can be calculated by utilizing Fig. 4. A current impulse is injected between the drain and source nodes of M5 as shown in Fig. 4. It is obvious to say that the applied current impulse just influences C1 and its voltage changes. As a result, the voltage across C1 is yielded as

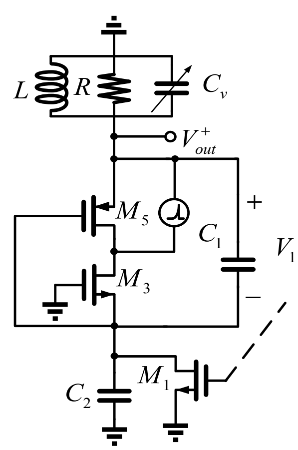

Fig. 4. Proposed the suitable half circuit to calculate Γ_{M5}

$$\Delta V_1 = (1-n)\Delta V_{out} \tag{8}$$

$$n = \frac{C_1}{C_1 + C_2}, (1-n) = \frac{C_2}{C_1 + C_2} \tag{9}$$

The voltage changes across the tank capacitance lead to the tank ISF. This means,

$$\Delta V_{out}^{+} \rightarrow \Gamma_R^{+} \tag{10}$$

Thus, the voltage changes across C1 results in

$$\Delta V_1 \rightarrow \Gamma_{M5} \tag{11}$$

Γ_{M5} can be written as

$$\Gamma_{M5} = (1-n)\frac{-\sin(\omega_0 t)}{N} \tag{12}$$

Γ_{M3} is the same as Γ_{M5} since M5 and M3 are series.

$$\Gamma_{M3} = (1-n)\frac{-\sin(\omega_0 t)}{N} \tag{13}$$

Fig. 5 is used to study the ISF of M1, Γ_{M1}. A current impulse is applied to the node a in Fig. 5. V_a can be associated to V_{out}^{+} as

$$V_a = nV_{out}^{+} \tag{14}$$

Consequently, Γ_{M1} is

$$\Gamma_{M1} = n\Gamma_R = n\frac{-\sin(\omega_0 t)}{N} \tag{15}$$

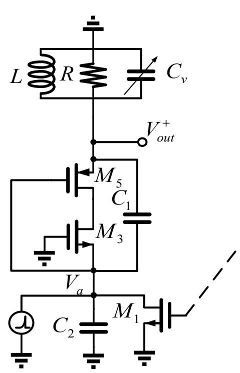

Fig. 5. The suitable half circuit to calculate Γ_{M1}

E. Noise Modulating Function (NMF)

As transistors have a cyclostationary noise, their noise modulating functions must be calculated. The drain noise current power of NMOS transistor can be expressed as [5]

$$\frac{\Delta \overline{i_n^2}}{\Delta f}(\omega_0 t) = 4KT\gamma g_m(\omega_0 t) = 4KT\gamma \frac{\partial id(\omega_0 t)}{\partial V_{gs}(\omega_0 t)} \tag{16}$$

Where K is Boltzmann's constant, T is the absolute temperature, ω_0 is the oscillation frequency, g_m is the transistor transconductance, and γ is the channel noise factor, which is commonly 2/3 in long -channel transistors. Because the drain current is periodic, it can be expressed with its Fourier series as

$$id(\omega_0 t) = \sum_{i=0}^{\infty} I_{i,d}\cos(i\omega_0 t + \phi_p) \tag{17}$$

$g(\omega_0 t)$ in (16) can be rewritten as

$$g(\omega_0 t) = \frac{\partial id(\omega_0 t)}{\partial Vgs(\omega_0 t)} = \frac{\frac{\partial id(\omega_0 t)}{\partial t}}{\frac{\partial V_{gs}(\omega_0 t)}{\partial t}} \tag{18}$$

Besides, since the tank Q is moderately high, the voltage nodes in an LC oscillator is nearly sinusoidal. Consequently, V_{sg5} can be written as

$$Vsg_5 = V_{dc} + A(1+n)\cos(\omega_0 t) \tag{19}$$

Where V_{g5} is $-nA$ [9]. Upon substituting (17) and (19) in (16), the drain noise current can be obtained as

$$\frac{\Delta \overline{i_n^2}}{\Delta f}(\omega_0 t) = 4KT\gamma \frac{-i\omega_0 \sum_{i=0}^{\infty} I_{i,out}\sin(i\omega_0 t + \phi p)}{-A(1+n)\omega_0 \sin(\omega_0 t)} \tag{20}$$

Using (5), (12), and (20), as well as supposed φ_P is zero and $\omega_0 t$ is φ, N_{M5} is

$$N_{M5} = 4KT\gamma \frac{1}{2\pi}\int_0^{2\pi} \frac{\sum_{i=0}^{\infty} iI_{i,d}\sin(i\omega_0 t)}{A(1+n)\sin(\omega_0 t)}\left(\frac{(1-n)^2}{N^2}\sin^2(\omega_0 t)\right)d\omega_0 t \tag{21}$$

$$N_{M5} = \frac{2KT(1-n)^2}{\pi AN^2(1+n)}\int_0^{2\pi}\left(\sum_{i=0}^{\infty} i\ I_{i,d}\sin(\phi)\right)\sin(\phi)d\phi \tag{22}$$

Note that the tank filters out the higher harmonics except for the first harmonic, therefore

$$N_{M5} = \frac{2KT(1-n)^2 I_{1,d}}{N^2 A(1+n)}\int_0^{2\pi}\sin^2(\phi)d\phi \tag{23}$$

$$N_{M5} = \frac{(2KT\gamma)}{N^2}\frac{(1-n)^2}{(1+n)}\left(\frac{I_{1,d}}{A}\right) \tag{24}$$

With a good approximation, $I_{1,d}/A$ is $1/R_{tank}$. Remember N is 2 since there are two resonators in the proposed oscillator.

$$N_{M5} = \frac{KT\gamma}{2}\frac{(1-n)^2}{(1+n)}\frac{1}{R_{\text{tank}}} \tag{25}$$

In the same manner, N_{M3} and N_{M1} can be obtained as

$$N_{M3} = \frac{KT\gamma}{2}\frac{(1-n)^2}{n}\frac{1}{R_{\text{tank}}} \tag{26}$$

$$N_{M1} = \frac{KT\gamma}{2}n\frac{1}{R_{\text{tank}}} \tag{27}$$

Also, the noise of tank resistance is stationary and $N_{R,tank}$ using (7) is yield as

$$N_{R,\tan k} = \frac{K_B T}{2}\frac{1}{R_{\text{tank}}} \tag{28}$$

F. Phase Noise

Replacing (28), (27), (26), (25), and (1) into (4), phase noise is obtained as

$$L(\Delta\omega) = 10\log\left(\frac{K_B T}{4\Delta\omega^2 C^2 I_B^2 R_{\text{tank}}^3}\left(\frac{1}{(2-n)^2} + \frac{\gamma n}{(2-n)^2} + \frac{\gamma(1-n)^2}{n(2-n)^2} + \frac{\gamma(1-n)^2}{(2-n)^2(1+n)}\right)\right) \tag{29}$$

Where in (29) supposed $\gamma_p \approx \gamma_n \approx \gamma$ and C is

$$C = C_v + \frac{C_1}{C_1 + C_2} \tag{30}$$

In (29), phase noise is clearly related to n, the sizes of C_1 and C_2. If n is optimized, the phase noise will be improved. When n is 0.46, 4 terms in the parenthesises have a minimum

III. SIMULATION

The simulation of ISFs of M1, M3, and M5 are illustrated in Fig. 6. As can be seen, the plots are of a $-sin(\varphi)$ trend showing a match with the theoretically derived ISFs in (12), (13), and (15). The discrepancies between the plots and the derived ISFs can be justified because the parasitic capacitances are ignored and it is well known every independent capacitance can affect ISFs. Basically, every single independent capacitance, no matter whether it is a parasitic capacitance or a capacitance purposefully designed in an oscillator by a designer can affect ISFs [5]. The simulated and theoretically derived phase noise is demonstrated in Fig. 7. The derived phase noise is compared with the simulated one in Fig. 7 representing a good accuracy between them so that the differences between them are less than 3 dB over the offset frequencies.

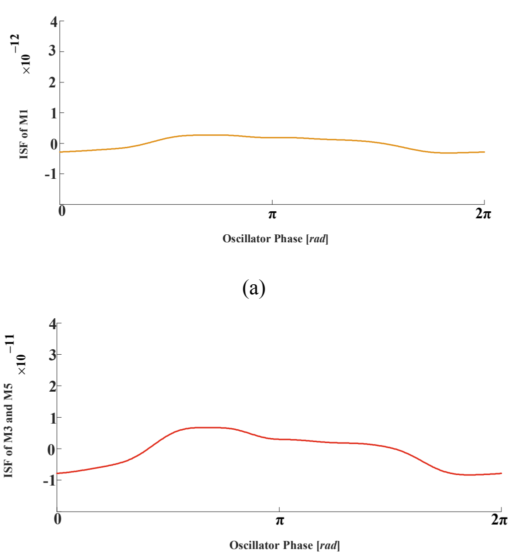

Fig. 6. The simulated ISFs of (a) M1 (b) M3 and M5 for thermal noise

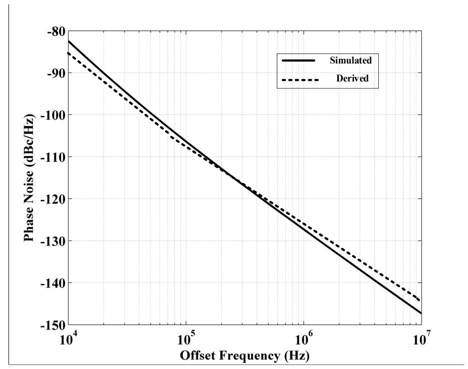

Fig. 7. The simulated and derived phase noise

IV. CONCLUSION

A comprehensive analytical phase noise study of a modified cross coupled oscillator is presented in this paper. To derive a closed-form formula of the phase noise in the $1/f^2$ region by Hajimiri's phase noise theory, the large signal oscillation amplitude is first provided and then the effective ISFs of the tank and transistors are calculated. Since the transistors have a cyclostationary noise, the noise modulating function is derived for them as well. The simulation of the ISFs shows they have a good precision between the simulations and the derived ones. In addition, the phase error between the simulated phase noise and the calculated one indicates less than 3 dB errors over the offset frequencies.

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