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Active Vibration Control for Building Structures based on $\mathcal{H}_\infty$ Synthesis Problem

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***Abstract*—This paper presents the development of an $\mathcal{H}_\infty$ controller synthesis designed based on the infinity norm and the frequency spectrum of the disturbance to achieve the reduction of vibrations in buildings structures subjected to seismic excitation. A feature of the proposed controller is that it can be designed around an experimental test. Through verifying Doyle's assumptions in control $\mathcal{H}_\infty$ theory ensure the stability of the closed-loop system. The mathematical model is introduced by means of mechanical elements and generalized through modal analysis. The effectiveness of the proposed controller is illustrated employing simulation results of a reduced scale two-stories building structure and the proposed algorithm is tested against the 1985's Mexico city earthquake data.**

***Index Terms*—Active control vibration, $\mathcal{H}_\infty$ controller, seismic excitation, building structure.**

I. Introduction

Engineering has always been concerned with counteracting the effect of earthquake vibrations and having structural control, for the countless lives lost, economic damage caused and paralysis of a nation [1]. There are different types of control according to the types of actuators used, for example, the classic dampers represent a passive control, control over the force exerted by these passive elements are called semi-active. In this context, reference [2] presents a state-of-the-art about this types of controllers, that are compared with passive and active controls. A more recent comparison is presented in [3], which denotes the great importance of vibration isolation in structures throughout the years. Active Mass Damper is the most popular techniques that has gained more interest for structural protection, due to the success obtained in its experimental development implementation [4]. Therefore, there are many control techniques in the literature such as a modified positive position feedback controller is proposed by [5], where the frequency of the controller is adapted to tracks the vibration frequencies through the fast Fourier transforms. Another design in optimal control forces for active tendons and active tuned mass dampers in order to introduce active damping to a tall building, even under nonlinear structural behavior is presented in [6]. In [7] a robust $\mathcal{H}_\infty$ controller design algorithms involving input constraints are carried out via linear matrix inequalities. Or classic PD-PID control algorithms merged with fuzzy logic to control non-linearity [8].

This paper aims to give an alternative approach to design a $\mathcal{H}_\infty$ controller, that is based on disturbance bandwidth reflection to improve the controller performance. A range of controller is determined by *hinfsyn* matlab ® function conditions. The effectiveness of the proposed control scheme is illustrated with simulation results of a reduced-scale two-storey building structure. The obtained results are compared with those from a PD controller, showing that the proposed approach has better performance and it is robust to measurement noise.

The remainder of the paper is organized as follows: Section II describes the dynamic mathematical model of the building structure. The development of that proposed PD controller is detail in Section III, whereas simulation results are given in Section IV and concluding remarks are provided in Section V. Finally, discussions and future works are presented in Section VI.

Notation: Throughout the paper the superscript $\|\cdot\|_\infty$ refers to the infinity norm, $\mathcal{S}$ is the set of transfer functions stable and proper and e exponent with base 10. The identity and null matrices of appropriate dimensions are represented by I and 0, respectively.

II. Mathematical model

Let the multi-degree of freedom building structure, subjected to horizontal seismic activity, whose dynamics is governed by

$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) + = -Ml\ddot{x}_g(t) \tag{1}$$

with

$$\begin{aligned} x(t) &= \left[x_1(t), x_2(t), \ldots, x_n(t)\right]^T \in \mathbb{R}^{n\times 1}, \\ \dot{x}(t) &= \left[\dot{x}_1(t), \dot{x}_2(t), \ldots, \dot{x}_n(t)\right]^T \in \mathbb{R}^{n\times 1}, \\ \ddot{x}(t) &= \left[\ddot{x}_1(t), \ddot{x}_2(t), \ldots, \ddot{x}_n(t)\right]^T \in \mathbb{R}^{n\times 1}, \\ l &= \left[1, 1, \ldots, 1\right]^T \in \mathbb{R}^{n\times 1}, \end{aligned}$$

where $x, \dot{x}, \ddot{x}$ are the vector of relative displacement, velocity and acceleration with respect to the basement; Vector l allows to distribute the signal $\ddot{x}_g$ in each storey. Moreover,

$M,\ D,$ and $K\ \in \mathbb{R}^{n\times n}$ are respectively the mass, damping, and stiffness matrix, defined as

$$M = diag\begin{bmatrix} m_1 & m_2 & \dots & m_n \end{bmatrix} > 0,$$

$$D = \begin{bmatrix} d_1+d_2 & -d_2 & \dots & 0 & 0 \\ -d_2 & d_2+d_3 & \dots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \dots & d_{n-1}+d_n & -d_n \\ 0 & 0 & \dots & -d_n & d_n \end{bmatrix} \geq 0,$$

$$K = \begin{bmatrix} k_1+k_2 & -k_2 & \dots & 0 & 0 \\ -k_2 & k_2+k_3 & \dots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \dots & k_{n-1}+k_n & -k_n \\ 0 & 0 & \dots & -k_n & k_n \end{bmatrix} > 0,$$

here $m_i,\ d_i,$ and k_i with $i = 1, 2, \dots, n$, are the *ith* mass, the inter-storey stiffness and damping, respectively, that define the physical characteristics of the building structure.

On the other hand, according to the modal decomposition theorem, vibration frequencies can be obtained with a linear superposition of vibration modes, which form a space base of deformed configurations of the structure [9]. Therefore,a displacement vector can be expressed in modal coordinates as:

$$x = \sum_{i=1}^{N} q_i \phi_i = \Phi q \tag{2}$$

with ϕ_i are the modes and

$$q_i(t) = e^{-\xi_i \omega_i t}\left[q_i(0)\cos(\omega_{iA}) + \frac{\dot{q}_i(0) + \xi_i \omega_i q_i(0)}{\omega_{iA}}\sin(\omega_{iA})\right]$$

modal coordinates for classical damping, where $\omega_{iA} = \omega_i\sqrt{1-\xi_i^2}$. Therefore, (1) can be defined in modal coordinates, as

$$M\Phi\ddot{q}(t) + D\Phi\dot{q}(t) + K\Phi q(t) = 0$$

pre-multiplying (II) by Φ^T

$$\Phi^T M\Phi\ddot{q}(t) + \Phi^T D\Phi\dot{q}(t) + \Phi^T K\Phi q(t) = 0$$

and defining $M_g = \Phi^T M \Phi$, $D_g = \Phi^T D \Phi$ and $K_g = \Phi^T K \Phi$, yields

$$M_g\ddot{q}(t) + D_g\dot{q}(t) + K_g q(t) = 0$$

with n decoupled differential equations. Moreover, using the generalized mass matrix, we obtain:

$$\ddot{q}(t) + 2\xi\omega_n\dot{q}(t) + \omega_n^2 q(t) = 0$$

where $\xi = [\xi_1, \dots, \xi_n]^T$ and $\omega_n = [\omega_{n_1}, \dots, \omega_{n_n}]^T$ vectors which depends on natural frequencies and damping factors.

Defining the state variables $x(t) = [x_1(t), x_2(t)]^T = [x(t), \dot{x}(t)]^T$, the system (1) can be written in the state space form as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -M^{-1}K & -M^{-1}D \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \underbrace{\begin{bmatrix} 0_{n\times 1} \\ l \end{bmatrix}}_{B_1} \ddot{x}_g(t) \tag{3}$$

$$y = \underbrace{\left[\{0\cdots 0\ 1\}_{1\times n}\ 0_{1\times n}\right]}_{C_2} x(t) \tag{4}$$

where the unique available output of the system is the displacement measured on the top floor. Note that the eigenvalues of matrix A always are complex conjugate poles, which can be expressed in terms of the natural frequencies and damping factors as [10]

$$s_{j,j+1} = -\omega_{n_j}\xi_j \pm i\ \omega_{n_j}\sqrt{1-\xi_j^2} \tag{5}$$

where $j \in \{1, \dots, 2n\}$ in such a way that each pair of poles allows obtaining the natural frequency and damping factor.

III. Controller design methodology

This section presents the development of a synthesis $\mathcal{H}_\infty$ controller, designed based on disturbance bandwidth, for structural vibration suppression purpose. To illustrate the methodology, the building structure is now equipped with an Active Mass Damper (AMD) on the roof, depicted in the Fig. 1.

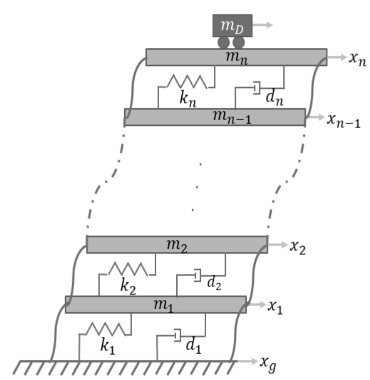


Fig. 1: Building with controller in the n-floor

The closed linear system with control action is now

$$M\ddot{x} + D\dot{x} + Kx = Ml\ddot{x}_g + \Gamma\left(u - f_d\right) \tag{6}$$

where u is the control signal, f_d the friction and damping force of the actuator and Γ is the location vector of the AMD actuator in the building, defined as follows

$$\Gamma_i = \begin{cases} 1 & \text{if } i = s \\ 0 & \text{otherwise} \end{cases},\ \forall\ i, \in \{1, \dots, n\},\ s \subseteq \{1, \dots, n\}$$

where s indicates the floors on which actuators of the active mass are installed. In this way, supposing that the AMD is located on the roof, then matrix Γ only have 1 in the position n. Accordingly, the effects of the actuator will be reflected in the state associated with the n-floor and Eq. (6) can be rewritten as:

$$\dot{x} = Ax + B_1\ddot{x}_g + B_2 u$$
$$y = C_2 x \qquad (7)$$

where A, B_1, C_2 are defined in the Eq. 3 and $B_2 = [\{0 \cdots 0\ 1/m_n\}_{1\times 2n}]^\intercal$ according with the actuator depicted in the figure 1.

Assumption 1. *The linear system can be stabilized by the $\mathcal{H}_\infty$ controller synthesis as a perturbations rejection, see Fig 2. The goal is to achieve $x_i(t) = \dot{x}_i(t) = 0$, $i = 1, 2, \ldots, n$.*

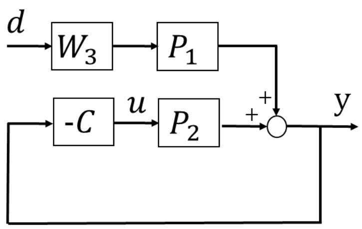


Fig. 2: $\mathcal{H}_\infty$ controller synthesis

where d a disturbance, P_1 is the transfer function between filtered d and y, P_2 is the plant between u and y calculated from (7), C is the controller and W_3 represent the weighted function of the disturbances, in this case, the seismic activity ($d = \ddot{x}_g$). On the other hand, P_2 for the $\mathcal{H}_\infty$ synthesis problem to be solvable is necessary two conditions: (A, B_2) must be stabilizable, and (A, C_2) must be detectable.

A. Design of weighted function (W_3)

To design of weighted function, the North-South component of the Mexico city 1985 earthquake measured at the Secretaría de Comunicaciones y Transportes (SCT) is used, whose frequency spectrum is compared with results reported in [11]. The weighted function is selected to match the raw earthquake data (disturbance) bandwidth in order to cancel it in the frequencies with most energy, capable of producing large displacements in the building structure. In this context, FFT allows to identify these frequencies that generally are in low frequencies and they are the most important for vibration control goal. Therefore, to overcome these problems, we propose a low-pass filter related with the disturbance bandwidth. Moreover, the filters performance are improved increase the order of them, for instance third-order, until achieving signals are maintained.

$$W_3 = \frac{1}{(\tau s + 1)^3} \qquad (8)$$

where $\tau = 0.05$ to match the frequency spectrum, as is depicted in Fig. 3.

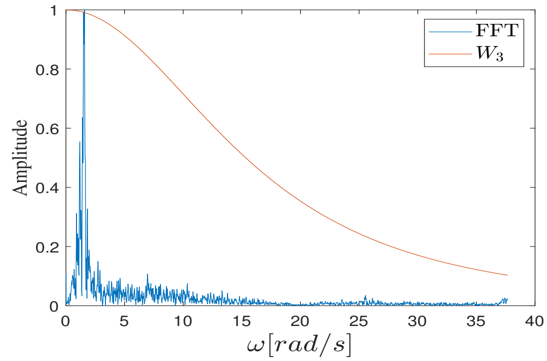


Fig. 3: FFT of 1985 earthquake data vs Weighted function W_3

B. Design of controller

The objective is find a controller C internally stabilizing, that minimizing $\|W_3 U\|_\infty$, where U is the transfer function that maps disturbance to output ($d \mapsto y$), see Fig 2:

$$U = \frac{P_1}{1 + P_2 C} \qquad (9)$$

The controller is based on the design for performance method [12], which can be summarized as:

Inputs: P_2, W_3

1) Realize a coprime factorization of P_2. Find four transfer functions in $\mathcal{S}$ satisfying::
$$P_2 = \frac{N}{M},\ NX + MY = 1 \qquad (10)$$
2) Find a stable function Q_{im} such that:
$$\|W_3 N(Y - NQ_{im})\|_\infty < 1. \qquad (11)$$
by using the solution of model-matching problem ($\|T_1 - T_2 Q_{im}\|$ with $T_1 = W_3 NY$ and $T2 = W_3 N^2$.
Obs.: U can be expressed in terms of a set a parameterized controllers (see step 5) as $U = N(Y - NQ)$.
3) Set
$$J(s) = \frac{1}{(\tau s + 1)^\nu} \qquad (12)$$
where ν is just large enough that $Q_{im}J$ is proper and τ is just small enough that
$$\|W_3 N(Y - NQ_{im}J)\|_\infty < 1.$$
4) Set $Q = Q_{im}J$
5) Set $C = (X + MQ)/(Y - NQ)$.

In this way, the controller is designed.

IV. Simulation results

In order to evaluate the performance of the proposed controller, a simulation of a reduce-scale two-storey building prototype are carried out. Parameters correspond to a mock up structure in the Department of Automatic Control at the Center for Research and Advanced Studies of the National Polytechnic Institute (CINVESTAV-IPN). This platform have masses of $m_1 = 3.17$ kg and $m_2 = 4.609$ kg, and stiffness

$k_1 = 9199.834$ N/m and $k_2 = 7531.628$ N/m. Moreover, since damping cannot be measured directly, this was approximate by means of Rayleigh model, yielding $c_1 = 7.388$ N s/m and $c_2 = 6.834$ N s/m.
Moreover, we consider the AMD is a linear servo actuator (STB1108, Copley Controls Corp.), that is mounted on the two storey. The moving mass of the damper weighs 5% of the mass of the building. The AMD is constrained to 90 N according to its datasheet specifications.
The simulations where carried out by means of Simulink ® of MatLab 2019b. The sampling time used was set to 0.001 s.

A. Analysis

Parameters m_i, k_i, and d_i with $i = 1, 2$ are replaced into the model (3) with null initial conditions of displacements and velocity. In addition, measurement errors are not considered in this work. The structural response is obtained after exciting the building structure by means of north-south component of 85's Mexico earthquake. For instance, Fig. 4 depict the building response at second storey[1].

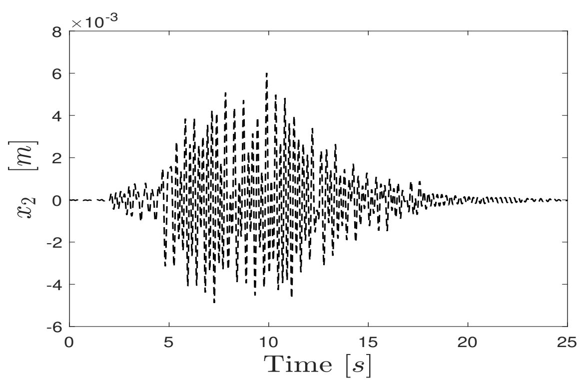


Fig. 4: Uncontrolled building response subject to 1985 earthquake excitation

Stability of the system is analyzed through pole placement $(Re(\lambda) < 0)$, that allows to calculate the region for the feedback control by root place. Details can be found in Fig. 5

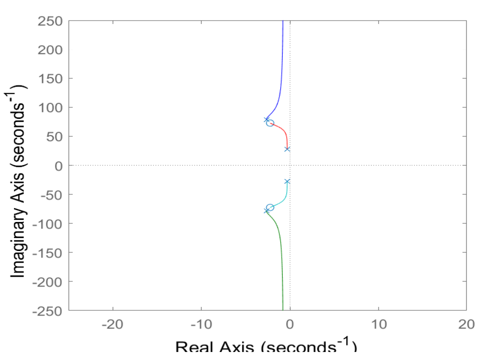


Fig. 5: Root locus

[1]Note that this figure is only as a reference signal to compare our results about controlled response, which are around 90 % smaller

that presents the eigenvalues of the open loop system through matrix A. The roots are located at (-2.6585 + 78.3114i, -2.6585 -78.3114i, -0.3261 + 27.7905i, -0.3261 -27.7905i), that are complex conjugate poles, which was to be expected due to matrices M, K and D are symmetric.

B. $\mathcal{H}_\infty$ controller design

The $\mathcal{H}_\infty$ controller was designed taking into account the pole placement and procedure described in the subsection III-B, yields the following controller:

$$C = \frac{1e8s^8 + 1.19e9s^7 + 1.39e12s^6 + 9.88e12s^5 + \cdots}{s^8 + 2e4s^7 + 2.21e5s^6 + 2.44e8s^5 + \cdots}$$
$$\frac{5.74e15s^4 + 1.69e16s^3 + 6.57e18s^2 + 7.69e18s + 2.25e21}{1.45e9s^4 + 8.26e11s^3 + 1.31e12s^2 + 5.01e14s + 2.04e7}$$

the response of the system defining this controller is exposed in the Fig 6, whereas Fig. 4 illustrated uncontrolled response, the comparison between the open and closed loop system based on $\|W_3 U\|_\infty < \gamma$ criteria shows that $\mathcal{H}_\infty$ controller produce a significant decrease in displacement around 90 % less.

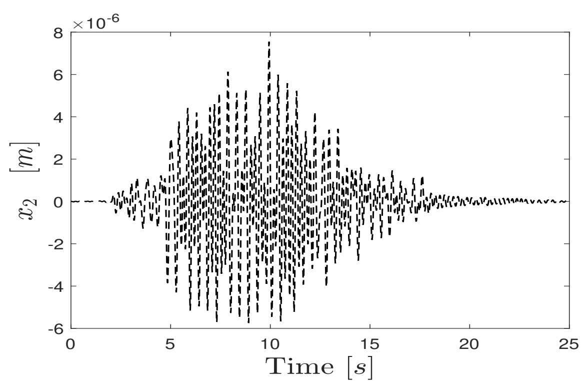


Fig. 6: Displacement of second floor with $\mathcal{H}_\infty$ controller under 1985 earth-quake excitation

C. Design of $\mathcal{H}_\infty$ controller by ***hinfsyn*** matlab function

Based in the section III [13], any system can be represented as:

$$\begin{aligned} \dot{x} &= Ax + B_1\ddot{x}_g + B_2u \\ z &= C_1x + D_{11}\varpi + D_{12}u \\ y &= C_2x + D_{21}\varpi + D_{22}u \end{aligned} \tag{13}$$

where $\varpi = \ddot{x}_g$ is the disturbance and z represents the error output, which should be minimized, indeed:

$$min \parallel z \parallel_2 \quad \equiv min \parallel T_{z\varpi} \parallel_\infty = \gamma$$

with the conditions over the plant, (A, B_2) must be stabilizable, and (A, C_2) must be detectable. The system (13) can be represented as matrix of transfer functions:

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} \varpi \\ u \end{bmatrix}$$

with (ϖ, u) as inputs, (z, y) as outputs and P the connection between them. Indeed, the weighted function W_3 can be introduced as follow

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} P_{11}W_3 & P_{12} \\ P_{21}W_3 & P_{22} \end{bmatrix} \begin{bmatrix} \varpi \\ u \end{bmatrix}$$

remembering that W_3 affects the disturbance, ϖ (cf. Fig. 2). Now, to apply this function with the example in its general form, from (3) two outputs are considered, hence C_2 is redefined as:

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Moreover, from (7) A, B_1 and B_2 are defined as:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -5278 & 2376 & -4.486 & 2.156 \\ 1634 & -1634 & 1.483 & -1.483 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.217 \end{bmatrix}$$

and $C_1 = C_2$ because the system use the same channel to express the effect of the disturbance. Moreover, before using this command, it is necessary to demonstrate the assumptions plant are true:

1) (A, B_2) stabilizable: $rank(\mathcal{C}) = size(\mathcal{C}) \rightarrow 4 = 4$.
2) (A, C_2) detectable: $rank(\mathcal{O}) = column\ size(\mathcal{O}) \rightarrow 4 = 4$.

The controller obtained with *hinfsyn* is represented by:
a) From output y_1 to signal control:

$$K_1(s) = \frac{-7.5068e07(s^2 + 124.2s + 4417)(s^2 + 58.04s + 4559)}{(s + 63.49)(s^2 + 73.24s + 4016)(s^2 + 5.317s + 6140)} \cdots \frac{(s^2 + 4.688s + 5722)(s^2 + 5.317s + 6140)}{(s^2 + 48.78s + 1.931e04)(s^2 + 627.9s + 1.86e05)}$$

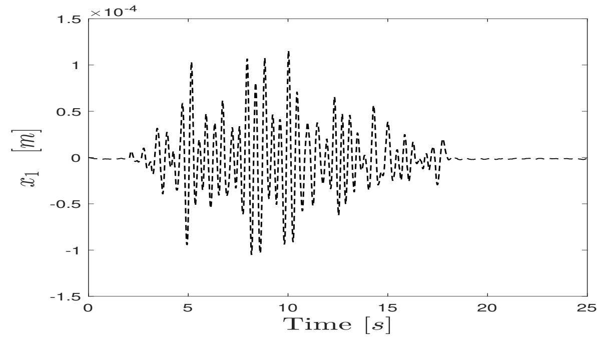
b) From output y_2 to signal control:

$$K_2(s) = \frac{-1.8679e07(s^2 + 108.5s + 3395)(s^2 + 50.54s + 3102)}{(s + 63.49)(s^2 + 73.24s + 4016)(s^2 + 5.317s + 6140)} \cdots \frac{(s^2 + 5.317s + 6140)(s^2 + 16.04s + 7530)}{(s^2 + 48.78s + 1.931e04)(s^2 + 627.9s + 1.86e05)}$$

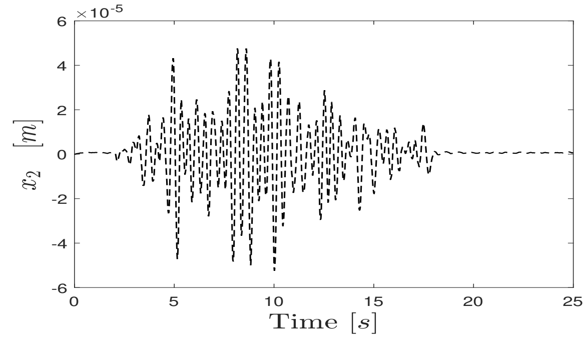
with $\gamma = 1.7277 \times 10^{-4}$.

The response of the each floor is showed in the Fig 7[2]. Test shown that the controller reduce displacements of each storey significantly 2 orders of magnitude. In conclusion, it is observed that proposed $\mathcal{H}_\infty$ controller has a favorable performance for vibration control and it is compared with the PD controller in the next section. Additionally, note that control variables was filtered with a low pass filter at 15 Hz, that corresponds to the bandwidth of the structural response. This eliminates sensor error as well as measurement noise in data. Moreover, offset was overcome adding a constant value to compensate it.

[2]Remark that Fig. 6 corresponds to $\mathcal{H}_\infty$ based on Doyle methodology, whereas 7b corresponds to hinfsyn matlab function, therefore, they are different displacements



(a)



(b)

Fig. 7: System response with $\mathcal{H}_\infty$ controller unser 1985 earthquake excitation (a) x_1 (b) x_2

D. PD Controller

The PD controller was designed to obtain a critically stable behavior and incremented the natural frequencies of the building. This natural frequencies calculated with the Eq. (5) and can be expressed as

$$\begin{aligned} \omega_{n_1} &= 78.35\ rad/s \quad \xi_1 = 0.03 \\ \omega_{n_2} &= 27.79\ rad/s \quad \xi_2 = 0.01 \end{aligned} \tag{14}$$

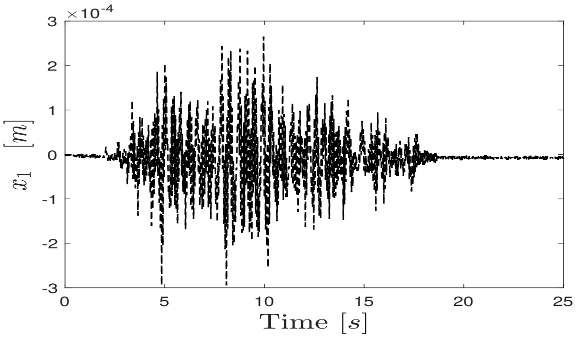
indeed, by pole placement the PD controller gains are obtained. The desired poles $(-219.89, -5.94, -3.83 + 72.82i, -3.83 - 72.82i)$ generate $K_d = 486.1169$ and $K_p = 894.7397$ gains, that produces the results shown in Figs. 8, that compared with $\mathcal{H}_\infty$ controller results in Fig (7), it is possible to see that it has a lower reduction, therefore, the $\mathcal{H}_\infty$ controller have advantages for vibration control purpose. A measure of system performance formed by integrating the square of the system error (ISE) over a fixed interval of time and is defined a

$$ISE = \int_t^T (\eta e)^2 d\tau \tag{15}$$

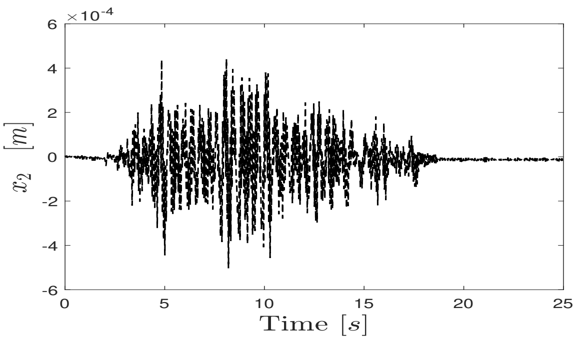
where η is a scaling factor and the error is defined as:

$$e = x_d - x$$

where $x_d = 0$ is the desired displacement, therefore, the displacement ISE for the top floor with $\eta = 1$ produce following error measures:



(a)



(b)

Fig. 8: System response with PD controller for 1985 earthquake excitation (a) x_1 (b) x_2

Controller	ISE
$\mathcal{H}_\infty$	2.2591×10^{-10}
hinfsyn function	1.1797×10^{-09}
PD	2.6146×10^{-07}

TABLE I: ISE of controllers

Consequently, the $\mathcal{H}_\infty$ controller shows better performance under this criterion.

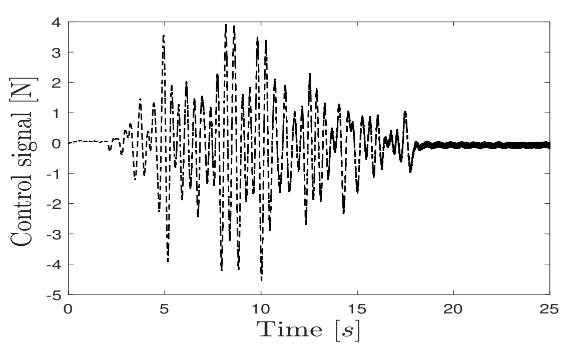


Fig. 9: Control signal seismic excitation

From Fig. 9 is evident that the control signal is below the maximum restriction of the actuator force 90 N, demonstrating its versatility for future practical applications.

V. CONCLUSION

In this paper, an $\mathcal{H}_\infty$ controller has been introduced in order to improve a building's response subjected to seismic events. $\|W_3U\|_\infty < \gamma$ is functional in setting limits of the controller. It showed a generalization for the design of vibration control in structures easy and effective. The methodology of $\mathcal{H}_\infty$ controller design considerably reduces displacements and avoids distortion between floors from coprime factorization and model-matching problem.

VI. DISCUSSION AND FUTURE WORKS

How reduce the order of the controller?. Is a mechanical system capable to achieve a signal control?
A future works design procedure from natural frequency of the buildings that are easy to measure experimentally and this leads to greater ease of tuning.

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