Charge of LiPo Batteries via Switched Saturated Super-Twisting Algorithm

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Abstract—The life of a LiPo battery depends on various factors such as charging methods, battery age, conditions of use, among others. Most of them are inevitable and challenging to prevent. However, the charging method can prolong the life span of the batteries, by guaranteeing that all the battery cells are charged to their full capacity. To achieve this objective a balanced load must be performed, such that the current supplied during the charging process follows a specific profile. In this work, a current supply control scheme is proposed for the LiPo battery charging stage, using a Boost type power converter. To ensure a specific current profile, the converter's current must follows a desired trajectory associated with cell balancing. The converter's control is done with a Saturated Super-Twisting Algorithm in its switched version and a Sigma-Delta Modulator. The proposed scheme guarantees the tracking of current in finite-time despite the presence of Lipschitz disturbances. Finally, the proposed strategy is validated by numerical simulation.

Index Terms—Power Converters, Sliding Mode Control, LiPo Batteries, Super-Twisting Algorithm, Sigma-Delta Modulator.

I. INTRODUCTION

Batteries have had a significant development in the last decades, e.g., the increase in discharge current and energy storage, as well as the reduction of their size. However, their lifetime is reduced due to many factors, like high temperatures or cell's voltage lower than 3.3 V, most of these factors occur while batteries are charging.

So there is a need to design LiPo battery chargers that are robust to exogenous disturbances to preserve battery lifetime. There are three different types of battery charging strategies [1]: 1. Constant current and constant voltage, 2. Current pulses, 3. Voltage pulses. In [2] a LiPo battery charger is designed and implemented in an FPGA (Field Programmable Gate Array). Also, [3] reports a functional prototype for small capacity batteries.

The DC-DC Boost power converter is widely used in battery charging when the supply voltage is less than the battery voltage level, e.g., in solar energy storage [4], [5]. In electric cars, the charging of batteries employing thermoelectric generation also uses a Boost type power converter [6], [7].

The first order sliding mode control strategy [8] has been used in power converters, by maintaining the intrinsic characteristic of the power converter, namely the control input belongs to a discrete set and making the system insensible to matched uncertainties/perturbations. However, this controller generates high level of chattering (ripple). In [9] higher order sliding mode controller were presented, where the order of the controller is chosen based on the relative degree of the controlled output. When this kind of controllers are used the resultant closed loop system can be seen as a relay system. The Super-Twisting Algorithm (STA) can be used for output with relative degree one. This controller make the system insensitive to matched uncertainties/perturbations and generates a continuous control signal, this last characteristic allows to diminish the chattering. However, it is not possible to guarantee that the control signal remains bounded. Recently in [10] a switched strategy that combines a STA an a first order sliding mode controller has been proposed. This strategy guarantees the compensation of matched uncertainties/perturbations for systems with relative degree one while guarantees the control signal remains bounded.

Moreover, for power converters, different control strategies have been developed. In [11] different controllers are proposed to the Boost converter in order to charge LiPo batteries under the presence of disturbances. In [12] a sliding mode controller is designed where the sliding surface is the current error, the current is estimated with an integral reconstructor. In [13] a Sigma-Delta Modulator ($\Sigma\Delta M$) with a passivity-based controller is implemented. In ([14], [15]) the control law
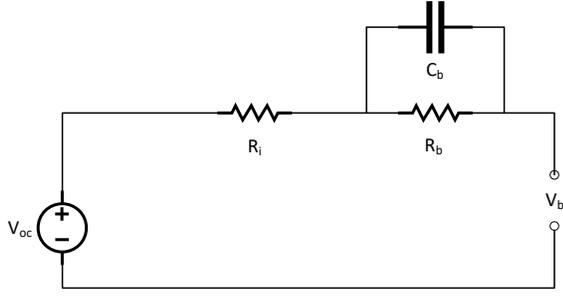
Fig. 1. Equivalent electric circuit of a LiPo battery.

Fig. 2. DC-DC Boost power converter.

Fig. 3. Charging process of a LiPo battery.

objective is solved through a differential flatness based controller where the flat output is the energy of the power converter. Different sliding mode controller implementation are presented in [16].

In this paper, the Saturated STA (SSTA) proposed in [10] is modified to use it properly in the power converter. This algorithm contains a state machine that switches between a STA controller and a relay function, such that when the STA control signal exceeds the limits of the control law, it switches to the relay function. To avoid overshoots when the controllers are switched, the disturbance is estimated to initialize the STA integrator at the value of the disturbance. Finally, a $\Sigma\Delta M$ is used to discretize the continuous signal from the SSTA controller.

The work is organized in the following form. The section II presents the preliminaries and the problem statement. In section III the control scheme is presented. Section IV presents some numerical results to validate the proposed strategy. Some conclusions are presented in section V.

II. PRELIMINARIES AND PROBLEM STATEMENT

In this section the LiPo battery model, the DC-DC Boost power converter and some useful results used on this work are presented. The problem statement is also given.

A. A LiPo Battery Model

Fig. 1 shows the equivalent electric circuit of a LiPo battery [17], [18]. The parallel configuration of the Capacitor C_b and the resistance R_b represents the storage capacity of the battery. R_i is the internal resistance and V_{oc} is the open-loop voltage source which depends on the state of charge (SoC).

In order to get the battery voltage, the open-loop voltage is computed. Applying the Thevenin theorem

$$V_b(t) = V_{oc} - R_i i_b(t) - R_b i_b(t) e^{\left(-\frac{t}{R_b C_b}\right)}, \qquad (1)$$

where $i_b(t)$ is the current of the mesh. The state of charge SoC is computed integrating the current flowing through the battery with the equation (see [18])

$$SoC = SoC_0 - \frac{1}{C_n} \int \frac{i_b}{3600} dt \qquad (2)$$

where SoC_0 is the initial state of charge, C_n denotes the battery capacity.
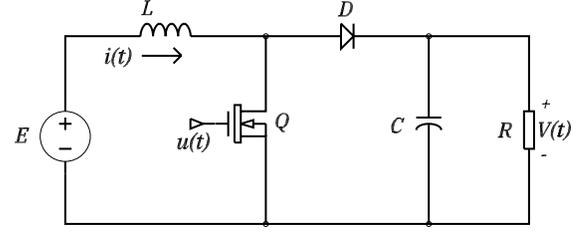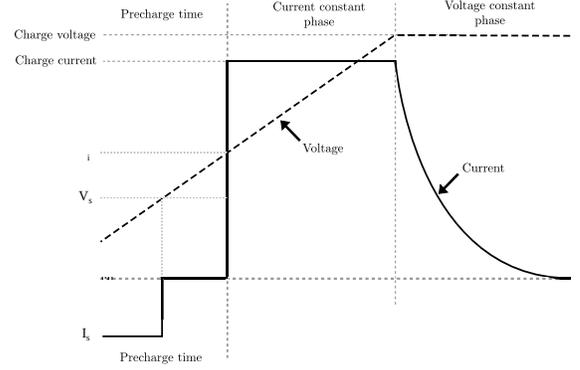
B. DC-DC Boost Power Converter Model

A DC-DC Boost power converter is proposed (see Fig. 2) to provide the required current during the charging stage. A Boost power converter (or step-up converter) has an output voltage $v(t)$ higher than the input voltage E. Assume the elements of the circuit are ideal, i.e., the switching speed of transistor Q is infinite, and the diode D does not consume energy. The converter is modeled as a bilinear system described by the following equations [19]:

$$\begin{aligned}
\frac{di(t)}{dt} &= -u_{av}(t)\frac{v(t)}{L} + \frac{E}{L} + \phi_1(t), \\
\frac{dv(t)}{dt} &= u_{av}(t)\frac{i(t)}{C} - \frac{v(t)}{CR},
\end{aligned} \qquad (3)$$

where i is the inductor current, v is the capacitor voltage and ϕ_1 represents the exogenous disturbances, unmodelled dynamics and energy losses as heat. The parameters L, C and R are the inductance, capacitance and load resistance respectively. The switched variable $u_{av}(t) = 1 - u(t)$ with $u(t) \in \{0,1\}$ is the control signal.

C. Charge Method Properties

The controlled charging methods aim for all cells to have the same voltage, Fig. 3 shows a method widely used in the literature [20]. In the pre-charge phase, it is checked if the cell's voltage is greater than or equal to V_l $(3.3V)$, then a current is applied until one of the cells reaches the charging voltage $(4.2V)$. Finally, the charging current exponentially decreases until it reaches the end current I_s. The current profile of this method is known as cell-balancing.

D. Problem Statement

Consider a LiPo battery is charged following a cell-balancing profile i_b. A Boost power converter supplies the current required during the charging stage. The cell-balancing profile is reached if the inductor current i of the converter follows a twice differentiable bounded desired current i_d associated with i_b. Suppose the Boost converter modeled by (3) is affected by the disturbances ϕ_1.

Along the paper the following assumptions are considered:

A1 i_d is a positive bounded function with bounded first and second time derivatives, i.e.,

$$\left| \frac{d^k i_d}{dt^k} \right| \leq \iota_k, k = 0, 1, 2.$$

with $\iota_k \in \mathbb{R}_+$

A2 The disturbance ϕ_1 is bounded and Lipschitz, i.e.,

$$p_0 \leq \phi_1 \leq p_1, \quad \left| \dot{\phi}_1 \right| \leq \phi_{max}$$

where $p_0, p_1 \in \mathbb{R}$ and $\phi_{max} \in \mathbb{R}_+$.

Note that in assumption A1 it is required that the desired current i_d be a sufficiently smooth function. However, normally this current is related with a piece-wise smooth function as is depicted in Fig. 7. To guarantee the fulfillment of assumption A1 the piece-wise function can be approximated by using a polynomial approximation in the non-differentiable points that guarantee the differentibility of the desired current.

It is required to design an open-loop control scheme for the LiPo battery charging process that ensures the tracking of the cell-balancing profile. A SSTA scheme is proposed to guarantees the finite-time tracking of the desired current i_d in the Boost converter (3) during the battery charging process despite the presence of disturbances ϕ_1. The continuous control signal generated by the SSTA must remain in the interval $[0, 1]$ to be supplied by modulation to the Boost converter.

III. THE CONTROL SCHEME TO THE CURRENT SUPPLYING

To solve the tracking current problem during the charging process, consider the scheme shown in Fig. 4. In the following subsections, each part of the proposed controller will be designed. The SSTA is adapted to ensure the control signal remains in the interval $[0, 1]$ to suffice the conditions of the binary set required for the Boost power converter.

A. Design of the control law

Let's consider the mathematical model of the Boost converter (3). Define the tracking error $s(t)$ as the difference of the actual current and desired current,

$$s(t) = i(t) - i_d(t).$$

Notice that if $s = 0$, implies $i = i_d$. Derivating

$$\dot{s}(t) = -(1 - u(t))\frac{v(t)}{L} + \frac{E}{L} + \phi_1(t) - \frac{di_d(t)}{dt} \quad (4)$$

Under the assumption $\dot{s}(t) = 0$, the equivalent control (*cf.* [21]) is

$$u_{eq}(t) = \frac{L}{v(t)}\left(\frac{v(t)}{L} - \frac{E}{L} + \frac{di_d(t)}{dt} - \phi_1(t) \right). \quad (5)$$

Hence, the dynamics on the sliding mode are given by

$$\frac{di(t)}{dt} = \frac{di_d(t)}{dt}. \quad (6)$$

Thus, if the sliding variable reaches the origin in finite-time, i.e. $s(t) = \dot{s}(t) = 0, \quad \forall t > t_\alpha$ where t_α is the reaching time, it is ensured that $i(t) = i_d(t)$ for all $t > t_\alpha$.

If the perturbation satisfies the assumption A2 with

$$p_0 = -\frac{E}{L} - \iota_1$$

$$p_1 = \frac{v_{max}}{L} - \frac{E}{L} + \iota_1, \quad (7)$$

where v_{max} is the maximum voltage supported by the converter. Then (5) is bounded in $[0, 1]$. The bounds are computed following a simple algebraic procedure.

To accomplish the input signal restriction $u \in \{0.1\}$, the controller proposed in [10] is adapted, using the state machine shown in the Fig. 5. The states of the state machine are defined as

$$\eta(t) = \begin{cases} 0 & \text{if} \quad |s(t)| > \delta \vee 0 < \bar{u}(t) < \rho, \quad \rho < 1, \\ 1 & \text{if} \quad |s(t)| \leq \delta. \end{cases} \quad (8)$$

where $\delta > 0$ is a sufficiently small value that defines a ball centered in the origin. So, the proposed controller is made up of

- $\Sigma\Delta M$ [22]

$$\begin{aligned} \dot{\zeta}(t) &= \bar{u}(t), \\ \dot{x}(t) &= u(t) = \frac{1}{2}\left(1 + \text{sign}(\sigma(t))\right), \quad (9) \\ \sigma(t) &= \zeta(t) - x(t), \end{aligned}$$

it transforms the continuous control signal $\bar{u}$ into a binary set that takes values in the domain required by the converter.

- SSTA controller

$$\bar{u}(t) = \begin{cases} -\frac{1}{2}\lceil s(t) \rfloor^0 + \frac{1}{2} & \text{if} \quad \eta(t) = 0, \\ \frac{L}{v(t)}\left(u_s(t) + u_n(t)\right) & \text{if} \quad \eta(t) = 1, \end{cases}$$

$$\begin{aligned} u_s(t) &= -k_1\lceil s(t) \rfloor^{\frac{1}{2}} + z(t) \\ \dot{z}(t) &= -k_2\lceil s(t) \rfloor^0, \quad z(t_s) = \hat{z}(t_s), \quad (10) \\ u_n(t) &= \frac{v(t)}{L} - \frac{E}{L} + \frac{di_d(t)}{dt}; \end{aligned}$$

where $\lceil \cdot \rfloor^q = |\cdot|^q \text{sign}(\cdot)$ and t_s is the time when the controller switches to STA mode and $\hat{z}$ is the estimate of the disturbance. The SSTA-based controller is in charge of achieving finite-time tracking of the desired current i_d.

- Perturbation estimator

$$\begin{aligned} \dot{\hat{e}}(t) &= \beta_1\lceil e_1(t) \rfloor^{\frac{1}{2}} - \hat{z}(t) + \frac{v(t)}{L}\bar{u}(t) - u_n(t), \\ \dot{\hat{z}}(t) &= -\beta_2\lceil e_1(t) \rfloor^0, \quad \hat{z}(0) = 0, \quad \hat{e}(0) = s(i(0)), \end{aligned} \quad (11)$$
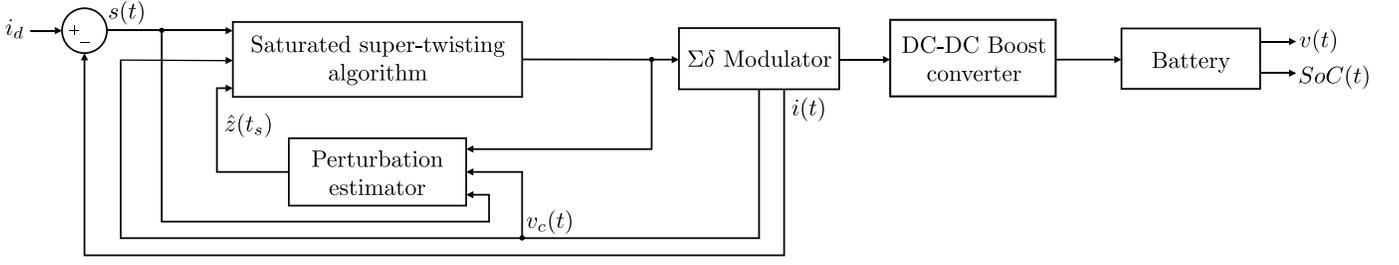
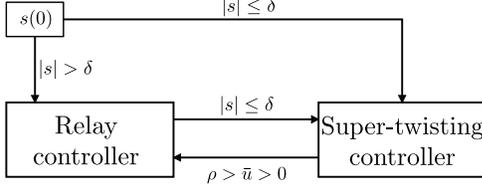Fig. 4. Open-loop scheme for the LiPo battery charging process.

Fig. 5. Diagram of the machine state.

Fig. 6. Response of the DC-DC Boost power converter to different initial conditions; (a) Current and (b) voltage when $u = 0$. (c) Current and (d) voltage when $u = 1$.

where $e_1(t) = s(t) - \hat{e}(t)$ with $\hat{e}$ the reconstruction of the sliding variable s, and e_1 the estimation error. System (11) avoids the Zeno phenomena in the machine state.

Note that if the initial condition of the error is outside the vicinity of the origin $|s(i(0))| > \delta$, then $\eta = 0$ so, there is a relay type behavior. If $|s(i(0))| \leq \delta$ the behavior is an STA, where it will be maintained until u leaves the defined bounds.

Now, the gains of the SSTA (10) and the estimator (11) are designed.

B. Closed-loop analysis

In this section, under the assumptions A1, A2, with bounds (7). A closed-loop analysis is performed to guarantee the stability of the system to any state of the state machine.

1) Relay case $\eta = 0$: The relay control saturates the output when the control law $\bar{u}$ (10) is outside the allowed limits and ensures the convergence of the error to a neighborhood of the origin.

If $s(t) > \delta$, i.e the current $i(t)$ is higher than the desired current $i_d(t)$, then the output of the relay is

$$\bar{u}(t) = -\frac{1}{2}\lceil s(t)\rfloor^0 + \frac{1}{2} = 0,$$

thus, $\dot{s}(t) < 0$ and s eventually will goes to the ball of radius δ.

If $s(t) < -\delta$, i.e., the current $i(t)$ is less than the desired current $i_d(t)$, the output of the relay is

$$\bar{u}(t) = -\frac{1}{2}\lceil s(t)\rfloor^0 + \frac{1}{2} = 1,$$

therefore $\dot{s}(t) > 0$ and s converges to the ball of radius δ.

The behavior of the converter for the nominal case, $\phi_1 = 0$, is shown in Fig. 6. In (a) and (b) the current and voltage are observed when $u = 0$ where the states converge to an equilibrium point, i.e., the system is stable. In (c) and (d) it is observed that the system is unstable. The voltage converges
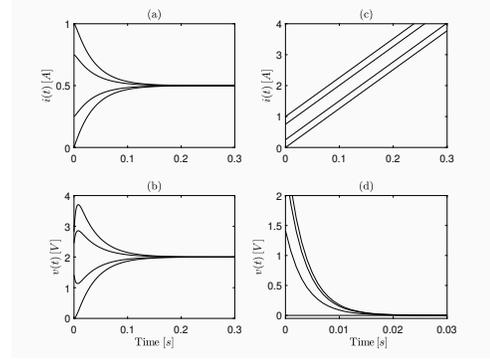
asymptotically to zero, but the current grows at a constant rate. The existence and uniqueness of the equilibrium point is given in [22].

Thus, the relay controller is capable to bring the sliding variable to the ball defined by $|s(t)| \leq \delta$ in finite-time t_δ if the disturbance is bounded by (7). The finite-time convergence of s to the ball can be proved by using a classical Lyapunov function $V(s) = \frac{1}{2}s^2$ where it is easy to show that

$$\dot{V}(s) \leq \frac{\sqrt{2}v_{max}V^{\frac{1}{2}}(s)}{L}.$$

Hence the sliding variable s converge to the ball $|s| < \delta$ at

$$t_\delta \leq \frac{2(\delta - |s(0)|)}{v_{max}}.$$

2) STA case, $\eta = 1$: In this case the control law is,

$$\bar{u}(t) = (-k_1\lceil s(t)\rfloor^{\frac{1}{2}} + z(t) + u_n(t))\left(\frac{L}{v(t)}\right)$$
$$\dot{z}(t) = -k_2\lceil s(t)\rfloor^0,$$

Differentiating the sliding variable,

$$\dot{s}(t) = -k_1\lceil s(t)\rfloor^{\frac{1}{2}} + z(t) + \phi_1(t). \tag{12}$$

Define $\Omega(t) = z(t) + \phi_1(t)$, then

$$\dot{s}(t) = -k_1\lceil s(t)\rfloor^{\frac{1}{2}} + \Omega(t),$$
$$\dot{\Omega}(t) = -k_2\lceil s(t)\rfloor^0 + \dot{\phi}_1(t).$$

If

$$k_1 = 1.5\sqrt{\phi_{max}}$$
$$k_2 = 1.1\phi_{max},$$

(13)

the convergence in finite-time of s and $\dot{s}$ to the origin is guarantee [23]. The proof that the switched strategy ensures finite-time convergence of s to the origin can be found in [24].

Observe that in case that the desired current is continuous but not differentiable in some measure zero points, the convergence of the SSTA cannot be guarantee and the controller must reconverge to the origin.

Note also, that the present closed loop analysis does not consider the $\Sigma\Delta M$. However, this element does not affect the convergence of the proposed controller since it translates the continuous control signal into a discontinuous one with the property that the equivalent output signal of the modulator matches the input signal generated by the SSTA controller (see [22] for more details).

C. Perturbation Estimator

When the controller change from the relay mode to the STA one, an overshoot may occur that cause the control signal to go out of the neighborhood close to the origin causing a Zeno behavior. The perturbation estimator ensures smoothness in the transition [10].

Assume that the controller starts in relay mode, i.e. $|s(0)| > \delta$ and that switches to STA mode in a minimum time $t_s > t_\delta$, where t_δ is the minimum time the converter remains in the relay mode. The disturbance estimator (11) is run in parallel so that at $t = t_s$ the STA integrator takes the value of the perturbation as initial condition.

Substituting (11) in the error dynamics e_1

$$\dot{e_1}(t) = -\beta_1\lceil e_1(t)\rfloor^{\frac{1}{2}} + \zeta(t)$$

where $\zeta(t) = \phi_1(t) + \hat{z}(t)$, then

$$\dot{e_1}(t) = -\beta_1\lceil e_1(t)\rfloor^{\frac{1}{2}} + \zeta(t)$$
$$\dot{\zeta}(t) = -\beta_2\lceil e_1(t)\rfloor^0 + \dot{\phi_1}(t)$$

(14)

If

$$\beta_1 = \begin{cases} \sqrt{8\beta_2} & \text{if } 0 \leq t \leq t_\delta, \\ 1.5\sqrt{\phi_{max}} & \text{if } t > t_\delta, \end{cases}$$

(15)

$$\beta_2 = \begin{cases} \phi_{max} + \frac{p_1}{t_\delta} & \text{if } 0 \leq t \leq t_\delta, \\ 1.1\phi_{max} & \text{if } t > t_\delta, \end{cases}$$

(16)

the disturbance ϕ_1 is estimated before $t = t_\delta$. The proof that these gains ensure convergence before $t = t_\delta$ can be found in [23], [25]. With this switched strategy, it is ensured that after the estimation error has converged to the origin, the gains are minimized to reduce the chattering without affecting the convergence.

IV. NUMERICAL RESULTS

The charge in a battery is simulated with parameters: $R_b = 0.015\,[\Omega]$, $C_b = 555\,[F]$, $R_i = 0.0033\,[\Omega]$. The initial condition of the state of charge is $SoC(0) = 0.001$. The parameters of the CD-CD Boost power converter are: $R = 4\,[\Omega]$, $L = 0.159\,[H]$, $C = 90\,[\mu F]$ and $E = 2[V]$.
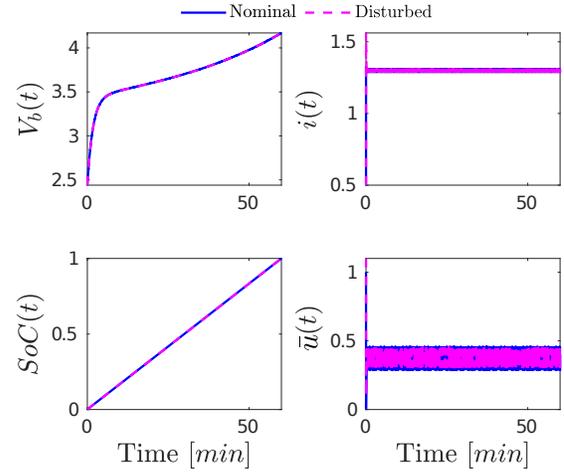

Fig. 7. Full charge of the LiPo battery.

The supply voltage must be lower than the minimum voltage of a LiPo cell $3.3\,[V]$. The initial conditions are $i(0) = 0.5\,[A]$ and $v(0) = 2\,[V]$. And the gains was selected as $k1 = 1.5\sqrt{\phi_{max}}$, $k2 = 1.1\phi_{max}$ with $\phi_{max} = 300$.

The open-loop charging battery strategy with the developed controller is shown in Fig. 4. The simulation of the full charge of a battery is shown in Fig. 7 where the sampling step is $0.01[s]$, and the following disturbance is presented throughout the simulation

$$\phi_1 = 0.1\sin(3\cos(4\pi t)\pi) + 0.2.$$

(17)

Observe that there is not any difference in the SoC during the charge in the presence of disturbances. Likewise, current and voltage are not affected by the disturbance. In Fig. 8 the transitory state of the charge is depicted, where the control input remains bounded in $\{0, 1\}$.

Fig. 9 shows the behavior of the Boost converter for a tracking task. The considered desired trajectory is

$$0.2\sin(4\pi t) + 1.3,$$

(18)

the magenta trajectories correspond to the case when there are perturbations (17) coupled to the control input. It is observed that the controller fulfills the tracking task in finite time so that the charger can follow a designed balancing-cell curve.

For comparison purposes consider a relay controller of the form

$$u(t) = \frac{1}{2} - \frac{1}{2}\text{sign}(s(t)).$$

(19)

The results obtained to the tracking task under the presence of disturbances is depicted in green in Fig. 9. Note that the relay controller presents more chattering than the proposed controller (10).

Notice that the proposed controller robustify the actuator of the charger, i.e., the Boost converter, but the charge of the battery is in open-loop. In order to close the loop of the battery charge is mandatory to design an observer to estimate the SoC, which is an open issue yet.
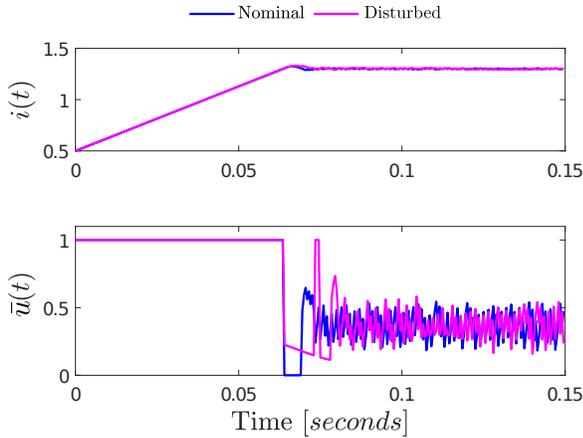
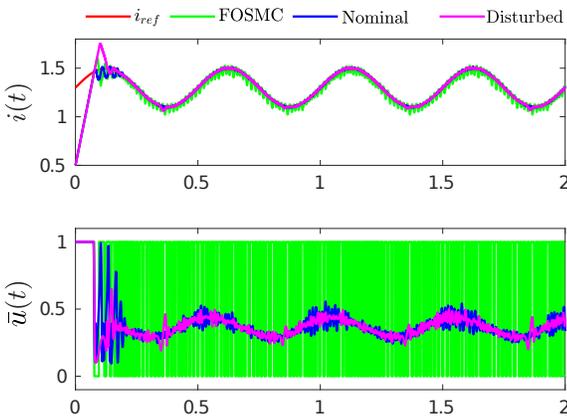Fig. 8. Transitory state of the full LiPo battery charge.

Fig. 9. Performance of the tracking with the controller proposed.

V. Conclusions

In this work, a current supply control scheme to the LiPo battery charging stage is presented. This methodology ensures that the inductor current of the Boost converter follows the desired path in finite-time, which is associated with a cell-balancing profile. The tracking is guaranteed in finite-time in the presence of matched Lipschitz disturbances. The proposed control signal was obtained using the SSTA algorithm in its switched version. To ensure that the converter receives a digital signal a $\Sigma\Delta M$ is used.

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